The Solution of Helmholtz Equation in Elliptical Domains

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The propagation of electromagnetic (EM) waves in hollow, perfectly conducting waveguides has been studied recently by Kretzschmar [1] in an excellent paper. Extensive numerical data are presented for the cutoff wavelength on 19 successive modes.

The paper is also of interest in other fields of applied sciences, since in the case of TM modes the problem is mathematically equivalent to that of an elliptical cylinder, subject to a sudden temperature change at the outer surface of the cylinder [2].

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The numerical evaluation of the solution requires the zeros of ordinary and modified Mathieu functions and the calculation of integrals involving the functions.

Kirkpatrick and Stokey's paper [2] describes also the evaluation of the temperature equation by the use of a digital computer giving results for ellipses having eccentricities of 0.60, 0.70, 0.80, and 0.90.

Kretzschmar [1] uses a Bessel-function product-series approach, while Kirkpatrick and Stokey [2] make use of a hyperbolic function series. It should also be pointed out that only the even TM_{cmp} $(m=0, 2, 4 \cdots, p=1, 2 \cdots)$ in Kretzschmar [1] have their equivalent in Kirkpatrick [2].

References

[1] J. G. Kretzschmar, "Wave propagation in hollow conducting elliptical waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 547–554, Sept. 1970.
[2] E. T. Kirkpatrick and W. F. Stokey, "Transient heat conduction in elliptica plates and cylinders," *J. Heat Transfer, Trans. ASME*, pp. 54–60, Feb. 1959

Letters

Instantaneous Frequency-Measuring Receivers

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The instantaneous frequency-measuring receiver (IFR) is becoming widely known to microwave systems engineers, but as a consequence of the defense applications its origins are obscure. The pioneering work on the phase-discriminator technique, which is the basis of most modern IFR systems, was done in the Mullard Research Laboratories, Redhill, Surrey, England. A brief account of the work and a record of one of the first signal observations with this type of receiver may be of interest.

Some of the unique characteristics of IFRs have already been detailed in the literature and are not repeated here. The IFR was perhaps one of the first significant postwar innovations in microwave receiver design. The principle for measuring frequency in terms of the phase delay of a signal propagated down a known length of transmission line must be nearly as old as electromagnetic science. However, the technique for applying this principle to broad-band receivers, whereby the frequency of any signal received in more than an octave range may be determined to an accuracy limited primarily by the signal duration, yet without the use of tuned filters or other circuit adjustments, is comparatively recent.

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Research towards this end was started in the Mullard Research Laboratories in 1954, and the first task was to broadband the hybrid junction, which was the building brick in many microwave circuits. Following a proposal by Hicken [1], and in collaboration with other workers, the phase-reversal hybrid was developed [2], [3]. In essence this was a proximity coupler inserted in one arm of an otherwise symmetrical four-port ring.

The application of this hybrid junction to broadband IFRs was recognized, but with a single junction used as a phase discriminator

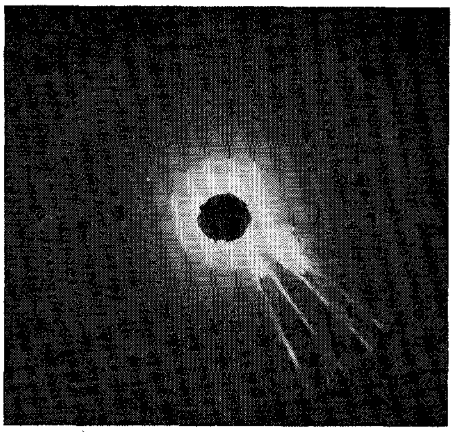

Fig. 1.

and with square-law detector diodes the detected signals at the output ports (with a delay-line length l) were unipolar and in the ratio $\tan^2(\pi lf/c)$. With the inevitable lack of amplitude balance and the internal mismatch reflections in a practical broad-band circuit, the relationship between frequency and any derived parameter was too grossly nonlinear and unrepeatable for accurate measurement.

A dramatic improvement in phase-discriminator design was made by S. J. Robinson of the Mullard Research Laboratories in 1957 [4], [5]. According to the simple trigonometric identity $\cos^2(\phi/2) - \sin^2(\phi/2) = \cos\phi$, subtraction of the detected junction outputs provided a bipolar voltage linearly related to the cosine of the phase and, therefore, the frequency. A second junction was coupled to the first with a net difference of $\pi/2$ between the respective phase delays and the outputs $\cos(2\pi lf/c)$ and $\sin(2\pi lf/c)$ applied to a cathode-ray tube indicator. The radial deflection was proportional to signal power and at an angle linearly related to frequency. Furthermore, the double-junction discriminator virtually eliminated the effects of amplitude unbalance and detector law, and substantially reduced the effects of internal mismatch reflections. For a given frequency range, say an octave, the phase delay line could be lengthened to give any number of 2π deflections around the CRT, i.e., any desired resolution. A combination of discriminators with different delay-line lengths provided a high-accuracy frequency indicator without ambiguity. For constant instrumental phase errors, the linearity of the frequency indicator improved in proportion to the phase delay. A number of IFRs using this discriminator technique were developed, and information on the receiver communicated to many other workers in the field.

A digital version of the receiver, in which only the polarities of the discriminator outputs were detected, was designed in 1960, and later versions were developed entirely in hybrid and monolithic integrated-circuit technique. R. N. Alcock, P. W. East, R. Levy, and J. L. Cook of the Mullard Research Laboratories participated in this development. Being, to a first order, independent of signal amplitudes, this receiver has a very large dynamic range. In its simplest form, a set of discriminators with phase delays in quaternary ratios provides a digital output in parallel bits with two bits per discriminator. The logic circuit for validating a pair of bits from the more significant pairs is simple and fast acting, and the output is in a convenient form for computer processing, Cartesian display, or digital control. Sophisticated frequency-filtering and signal cross-checking functions may be performed within the logic circuits [6].

In conclusion it may be of interest to illustrate a signal display recorded in 1958 which is believed to be the first observation of signals with a balanced phase discriminator IFR (Fig. 1). The receiver operated over an octave band from 2 to 4 GHz with one discriminator circuit. It is measuring five pulsed signals, one of which is a multifrequency transmission.

References

[1] E. M. Hicken and General Electric Co., U. K. Patent 770317, 1955.
[2] N. E. Goddard and S. J. Robinson, U. K. Patent 818018, 1957.
[3] S. J. Robinson, "Broad-band hybrid junctions," *IRE Trans. Microwave Theory Tech.* (Corresp.), vol. MTT-8, pp. 671–672, Nov. 1960.
[4] ——, U. K. Patent 953430, 1958.
[5] ——, "Comment on 'Broadband microwave discriminators'," *IEEE Trans. Microwave Theory Tech.* (Corresp.), vol. MTT-12, pp. 255–256, Mar. 1964.
[6] A derivative of the digital phase-measuring technique is also used for bearing angle measurements. See, for example, R. N. Alcock, D. Atter, S. J. Robinson, and R. P. Vincent, in *Proc. AGARD Conf. Helicopter Guidance and Control Systems*, Konstanz, Germany, June 1971; also NATO Advisory Group for Aerospace Research, CP-86-71, Neuilly-sur-Seine, France; also R. N. Alcock, *Philips Tech. Rev.*, vol. 28, p. 226, 1967.

Correction to "Distortion Performance of the Abrupt-Junction Current-Pumped Varactor Frequency Converter"

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In the above paper,[1] on page 746, Fig. 5 should appear as follows:

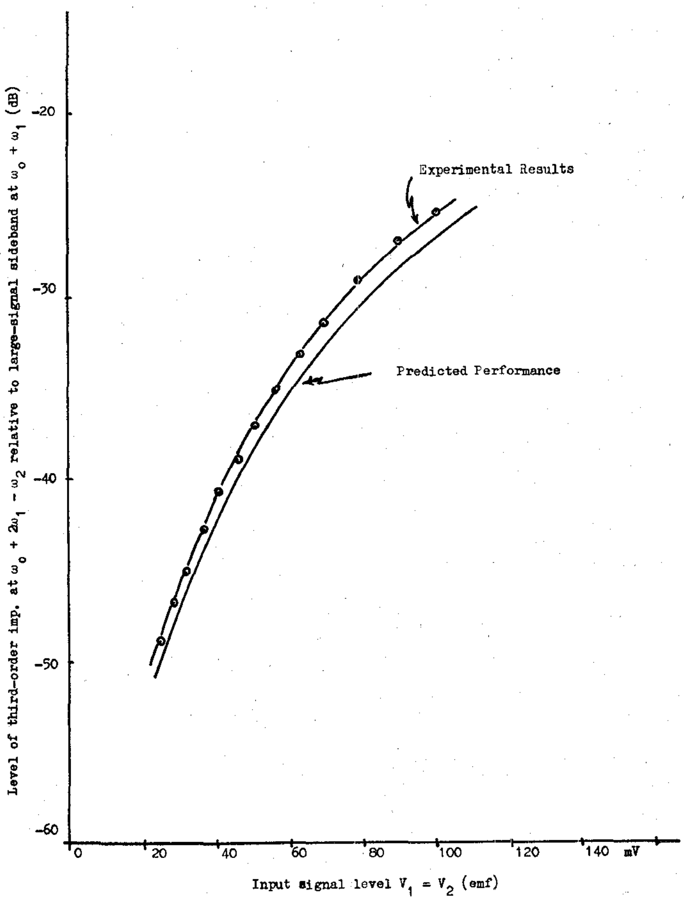

Fig. 5. Predicted and measured performance of abrupt-junction current-pumped converter.

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[1] J. G. Gardiner and S. I. Ghobrial, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 741–749, Sept. 1971.